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EVOLUTION OF THE HEAT TRANSFER AND FLOW IN MODERATELY SHORT TURBULENT BOUNDARY LAYERS IN SEVERE PRESSURE GRADIENTS

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Abstract—The early and intermediate development of highly accelerated velocity and thermal boundary layers are analyzed. For sufficiently large accelerations (or pressure gradients) and for total normal strains which are not excessive, the equations for the Reynolds shear stress and turbulent heat transfer simplify to give stresses and fluxes that remain approximately constant as they are convected along stream lines. The theoretical results for the evolution of Stanton number and of mean velocity and temperature distributions in favorable pressure gradients agree with experiment for the cases considered.

NOMENCLATURE

C_f, skin friction coefficient, $2\tau_w/(\rho U_\infty^2)$;
c_p, specific heat at constant pressure;
h, heat transfer coefficient, $q_w/(T_w - T_\infty)$;
K, pressure gradient or acceleration parameter, $-(\nu/\rho U_\infty^3)\,dP/dx_1 = (\nu/U_\infty^2)\,dU_\infty/dx_1$;
K_m, maximum K for a particular run;
P, mean pressure;
Pr, Prandtl number, ν/α;
p, fluctuating pressure component;
q_w, wall heat transfer per unit area;
R_j, defined by (14);
r_k, component of vector extending from point P' to P'';
r_k^*, dimensionless r_k, $U_0 r_k/\nu$;
St, Stanton number, $h/(\rho c_p U_\infty)$;
T, temperature;
T_w, wall temperature;
$(T_w)_0$, wall temperature at initial station;
T_∞, free stream temperature;
T^*, dimensionless temperature defined by (5);
T^+, temperature parameter, $(T_w - T)c_p\tau_w/[q_w\sqrt{(\tau_w/\rho)}]$;
t, time;
U_i, mean velocity component;
U_i^*, dimensionless mean velocity component U_i/U_0;
U_∞, free stream velocity;
U_0, free stream velocity at initial station;
$(U_1)_0$, mean longitudinal velocity along same stream line where U_1 is measured, but at initial station;
U_1^0, $U_1/(U_1)_0$;
U_1^+, velocity parameter, $U_1/\sqrt{(\tau_w/\rho)}$;
u_i, fluctuating velocity component;
$\overline{u_1u_2}$, turbulent shear stress;
$\overline{u_1u_2^*}$, dimensionless turbulent shear stress defined by (11);
x_i, space coordinate;
x_1^*, dimensionless space coordinate defined by (6);
x_0, value of x_1 at initial station;
x_2^+, dimensionless wall distance, $\sqrt{(\tau_w/\rho)}x_2/\nu$;
α, thermal diffusivity;
ν, kinematic viscosity;
ψ, stream function defined by (20);
ψ^*, dimensionless stream function, ψ/ν;
ρ, density;
τ, fluctuating temperature component;
$\overline{\tau u_2}$, turbulent heat transfer;
$\overline{\tau u_2}^*$, dimensionless turbulent heat transfer defined by (7);
τ_w, wall shear stress.

Superscripts
$'$, $''$, at points P' and P'';
$*$, on quantity non-dimensionalized by suitable combinations of U_0, $(T_w)_0 - T_\infty$, ν and ρ;
$\overline{}$, an averaged quantity.

INTRODUCTION

IN [1] WE analyzed a moderately short turbulent boundary layer in a severe pressure gradient as an initial value problem. The mean velocity profile and turbulence quantities were considered as known at an initial longitudinal position. For sufficiently large accelerations (or pressure gradients) and for total normal strains which were not excessive, the equations for the Reynolds shear stress simplified to give a stress that remains approximately constant as it is convected along stream lines. The evolution of the velocity boundary layer could then be calculated. The experiments of Blackwelder and Kovasznay [2] suggest the validity of the simplification used in [1], since although the pressure gradients caused the mean flow in those experiments to change considerably, the Reynolds stresses, at least in the important intermediate region of wall distances, were relatively unaffected.

Herein we carry out a similar analysis for the evolution of a thermal boundary layer. For that problem a simplification of the equations of motion and heat transfer allows us to write the turbulent heat transfer, as well as the Reynolds shear stress, as approximately constant along a stream line. As in [1], the method is applicable when the total longitudinal strain is not excessive and a pressure-gradient parameter is sufficiently large. Thus, although the present problem might seem at first to be more complicated than say the fully developed problem, it turns out to be relatively tractable within the framework of the present simplification.

Experimental studies of turbulent heat transfer in severe pressure gradients have been made by several investigators, e.g. [3–5]. In the analytical area, use has been made of semiempirical mixing-length and single-point two-equation models of turbulence [6–8]. The present analysis, on the other hand, formulates an initial-value problem which does not include adjustable constants or functions. The region considered is mainly the so-called relaminarization region for severe favorable pressure gradients which has been observed in the experiments of [3–5].

ANALYSIS

The equations for the development of the thermal boundary layer will be considered first; the equations for the velocity boundary layer were formulated in [1]. The equation for the mean temperature in an incompressible turbulent flow [9] is

$$\frac{\partial T}{\partial t} + U_k \frac{\partial T}{\partial x_k} = \frac{\partial}{\partial x_k}\left(\alpha \frac{\partial T}{\partial x_k} - \overline{\tau u_k}\right) \tag{1}$$

where T is the mean temperature, τ is the turbulent temperature component, U_k is a mean velocity component, u_k is a turbulent velocity component, x_k is a space coordinate, t is the time, and α is the thermal diffusivity. The overbar designates an averaged quantity, and a repeated subscript in a term indicates a summation of terms with the subscript successively taking on the values 1, 2, and 3. For a thin steady-state two-dimensional boundary layer, equation (1) becomes

$$U_1 \frac{\partial T}{\partial x_1} = -U_2 \frac{\partial T}{\partial x_2} + \alpha \frac{\partial^2 T}{\partial x_2^2} - \frac{\partial}{\partial x_2} \overline{\tau u_2} \tag{2}$$

where x_1 is in the direction of the main flow.

Equation (2) can be written in dimensionless form as

$$U_1^* \frac{\partial T^*}{\partial x_1^*} = -U_2^* \frac{\partial T^*}{\partial x_2^*} + \frac{1}{Pr}\frac{\partial^2 T^*}{\partial x_2^{*2}} + \frac{\partial \overline{\tau u_2}^*}{\partial x_2^*} \tag{3}$$

where

$$U_i^* = U_i/U_0 \tag{4}$$

$$T^* = (T_w - T)/[(T_w)_0 - T_\infty] \tag{5}$$

$$x_i^* = (x_i - x_0)U_0/\nu \tag{6}$$

$$\overline{\tau u_2}^* = \overline{\tau u_2}/[(T_w)_0 - T_\infty]U_0 \tag{7}$$

$$Pr = \nu/\alpha \tag{8}$$

and U_0 is the velocity outside the boundary layer at the initial station, $(T_w)_0$ is the wall temperature at the initial station, T_∞ is the constant temperature outside the thermal boundary layer, and ν is the kinematic viscosity. The dimensionless mean velocity components U_1^* and U_2^* are obtained in [1] from the momentum and continuity equations as

$$\frac{1}{2}\frac{\partial U_1^{*2}}{\partial x_1^*} = -U_2^* \frac{\partial U_1^*}{\partial x_2^*} - \frac{\nu}{\rho U_0^3}\frac{dP}{dx_1} + \frac{\partial^2 U_1^*}{\partial x_2^{*2}} - \frac{\partial \overline{u_1 u_2}^*}{\partial x_2^*} \tag{9}$$

and

$$\frac{\partial U_2^*}{\partial x_2^*} = -\frac{\partial U_1^*}{\partial x_1^*} \tag{10}$$

where

$$\overline{u_i u_j}^* = \overline{u_i u_j}/U_0^2 \tag{11}$$

and P, which is a function only of x_1, is the mean pressure.

In order to solve equations (3)–(11) to obtain the evolution of T^* and U_1^*, the Reynolds shear stress $\overline{u_1 u_2}$ and the turbulent heat transfer $\overline{\tau u_2}$ must be known at each point in the flow. Since the Reynolds shear stress was considered in [1], we will confine most of the present discussion to the turbulent heat transfer. The full two-point equations for the turbulent heat transfer are given in [10], and can be written in abbreviated dimensionless form for the steady-state case as

$$\begin{aligned}\frac{1}{2}(U_1'^* + U_1''^*)\frac{\partial \overline{\tau' u_j''}^*}{\partial x_1^*} &= -\frac{1}{2}(U_2'^* + U_2''^*)\frac{\partial \overline{\tau' u_j''}^*}{\partial x_2^*} \\ &\quad - \overline{u_k' u_j''}^* \frac{\partial T'^*}{\partial x_k'^*} - \overline{\tau' u_k''}^* \frac{\partial U_j''^*}{\partial x_k''^*} - \frac{1}{2}\frac{\partial \overline{\tau' u_k' u_j''}^*}{\partial x_k^*} \\ &\quad + \cdots - \frac{\partial \overline{\tau' p''}^*}{\partial r_j^*} + \cdots + \frac{1}{4}\left(1 + \frac{1}{Pr}\right)\frac{\partial^2 \overline{\tau' u_j''}^*}{\partial x_k^* \partial x_k^*} \\ &\quad + \left(1 + \frac{1}{Pr}\right)\frac{\partial^2 \overline{\tau' u_j''}^*}{\partial r_k^* \partial r_k^*}\end{aligned} \tag{12}$$

$$\frac{1}{4}\frac{\partial^2\overline{\tau' p''}^*}{\partial x_j^* \partial x_j^*}+\cdots = -2\frac{\partial U_j''^*}{\partial x_k''^*}\left(\frac{1}{2}\frac{\partial \overline{\tau' u_k''}^*}{\partial x_j^*}+\frac{\partial \overline{\tau' u_k''}^*}{\partial r_j^*}\right) - \frac{1}{4}\frac{\partial^2 \overline{\tau' u_j'' u_k''}^*}{\partial x_j^* \partial x_k^*}+\cdots \quad (13)$$

where, as in equation (3), the starred quantities have been non-dimensionalized by suitable combinations of U_0, $(T_w)_0 - T_\infty$, ν and ρ, and p is the fluctuating pressure component. The primes and double primes designate quantities at points P' and P'', which are separated by the vector $\boldsymbol{r}$, and the unprimed quantities are measured at x_i, which lies halfway between P' and P''.

Equations (12) and (13) contain triple velocity–temperature correlations which can be obtained from three-point equations. The latter contain quadruple correlations which in turn can be obtained from four-point equations, and so on. Thus, as in the case of the equations for the velocity correlations in [1], an infinite hierarchy of equations results. Equation (12) for $r_i = 0$ becomes

$$\frac{\partial \overline{\tau u_j}^*}{\partial x_1^*} = -\frac{U_2^*}{U_1^*}\frac{\partial \overline{\tau u_j}^*}{\partial x_2^*} + R_j \quad (14)$$

where R_j represents the remainder of the terms in equation (12) (for $r_i = 0$).

Each of the infinite hierarchy of higher order equations can be put into a dimensionless form similar to that of equations (3)–(14), or of equations (4)–(8) in [1]. Thus, for given initial flow and temperature fields, the flow and temperature fields at any position along the boundary layers are given by the functional equations

$$U_1^* = f_1[x_1^*, x_2^*, (\nu/\rho U_0^3)\,\mathrm{d}P/\mathrm{d}x_1] \quad (15)$$

$$\overline{u_i u_j}^* = f_2[x_1^*, x_2^*, (\nu/\rho U_0^3)\,\mathrm{d}P/\mathrm{d}x_1] \quad (16)$$

$$T^* = f_3[x_1^*, x_2^*, (\nu/\rho U_0^3)\,\mathrm{d}P/\mathrm{d}x_1] \quad (17)$$

$$\overline{\tau u_j}^* = f_4[x_1^*, x_2^*, (\nu/\rho U_0^3)\,\mathrm{d}P/\mathrm{d}x_1] \quad (18)$$

and an infinite hierarchy of similar equations for other turbulence quantities.

New parameters obtained by operating on the parameters in equations (15)–(18) can, of course, be used in their place, so long as the same total number is maintained. Thus, by using the relation

$$-\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}x_1} = U_\infty \frac{\mathrm{d}U_\infty}{\mathrm{d}x_1} \quad (19)$$

where U_∞ is the velocity at the edge of the boundary layer, it is shown in [1], that one of the original parameters can be replaced by U_∞/U_0.

Equations (3), (9), and (14) can be transformed from (x_1, x_2) to (x_1, ψ) coordinates (von Mises coordinates), where the stream function ψ is given by

$$\frac{\partial \psi}{\partial x_1} = -U_2, \quad \frac{\partial \psi}{\partial x_2} = U_1. \quad (20)$$

The result for $j = 2$ is

$$\left(\frac{\partial T^*}{\partial x_1^*}\right)_\psi = \frac{1}{Pr}\frac{\partial}{\partial \psi^*}\left(U_1^* \frac{\partial T^*}{\partial \psi^*}\right) + \frac{\partial \overline{\tau u_2}^*}{\partial \psi^*} \quad (21)$$

$$\frac{1}{2}\left(\frac{\partial U_1^{*2}}{\partial x_1^*}\right)_\psi = -\frac{\nu}{\rho U_0^3}\frac{\mathrm{d}P}{\mathrm{d}x_1} + \tfrac{1}{2}U_1^* \frac{\partial^2 U_1^{*2}}{\partial \psi^{*2}} - U_1^* \frac{\partial \overline{u_1 u_2}^*}{\partial \psi^*} \quad (22)$$

$$\left(\frac{\partial \overline{\tau u_2}^*}{\partial x_1^*}\right)_\psi = R_2 \quad (23)$$

where

$$\psi^* = \psi/\nu. \quad (24)$$

Equation (23) can be integrated along a stream line to give

$$\overline{\tau u_2}^* = (\overline{\tau u_2}^*)_0 + \int_1^{U_1^0} \frac{R_2 (\mathrm{d}U_1^0)_\psi}{[\nu/U_0(U_1)_0][(\partial U_1/\partial x_1)]_\psi} \quad (25)$$

where $U_1^0 = U_1/(U_1)_0$. The quantities $(U_1)_0$ and $(\overline{\tau u_2}^*)_0$ are, respectively, the values of U_1 and $\overline{\tau u_2}^*$ on the same streamline as U_1 and $\overline{\tau u_2}^*$ but at the initial station, and U_0 is the value of U_∞ at the initial station. The subscript ψ indicates changes along a streamline. The quantity R_2 will, at least initially, be close to zero, since for the initial zero-pressure-gradient boundary layer, the production terms are in approximate equilibrium with the other terms in R_2. Then if R_2 is not a strong function of $\partial U_1/\partial x_1$, equation (25) reduces to

$$\overline{\tau u_2} \approx \overline{\tau u_2}(\psi) = [\overline{\tau u_2}(\psi)]_0 \quad (26)$$

for sufficiently large $[\nu/U_0(U_1)_0](\partial U_1/\partial x_1)_\psi$ (or large $\nu/\rho U_0^3\,\mathrm{d}P/\mathrm{d}x_1$), and/or for $U_1/(U_1)_0$ (or U_∞/U_0) sufficiently close to one. A similar argument for $\overline{u_1 u_2}$ in [1] shows that for the same conditions

$$\overline{u_1 u_2} \approx \overline{u_1 u_2}(\psi) = [\overline{u_1 u_2}(\psi)]_0. \quad (27)$$

That is, if the pressure-gradient parameter (or the acceleration along stream lines) is sufficiently large, and the total normal strain $\ln(U_\infty/U_0)$ is not excessive, the turbulent heat transfer and Reynolds shear stress might be considered as frozen at their initial values as they are convected along streamlines. In that case $\overline{\tau u_2}$ and $\overline{u_1 u_2}$ for a particular flow are, of course, functions only of ψ.

Equation (25) and equation (16) in [1] indicate that the allowable values of the parameters are interdependent. For instance for $U_1/(U_1)_0$ (or U_∞/U_0) quite close to one, equations (26) and (27) may apply reasonably well even when the pressure gradient parameter $\{\text{or } [\nu/U_0(U_1)_0]\,\partial U_1/\partial x_1\}$ is moderately small. Conversely, for very large pressure gradient parameters $\{\text{or large } [\nu/U_0(U_1)_0]\partial U_1/\partial x_1\}$ it may be allowable to have relatively large departures of $U_1/(U_1)_0$ (or of U_∞/U_0) from one.

In order to get an idea of how important an effect normal strains might have on $\overline{u_1u_2}$, analytical results for locally homogeneous turbulence with uniform normal strain and shear without turbulence self-interaction are considered in [1]. Results for $(\partial U_1/\partial x_2)/(\partial U_1/\partial x_1) = 2$ indicate that for values of $U_1/(U_1)_0$ of 2 or less, $\overline{u_1u_2}$ should not vary more than about 14 per cent. For larger values of $U_1/(U_1)_0$ the variation of $\overline{u_1u_2}$ is somewhat greater. However, the variation of $\overline{u_1u_2}$ in an actual boundary layer, where the turbulence is inhomogeneous and the parameter $(\partial U_1/\partial x_2)/(\partial U_1/\partial x_1)$ is usually much greater than 2, appears to be somewhat less according to the experimental results of [2]. It might be expected that similar results would apply to $\overline{\tau u_2}$.

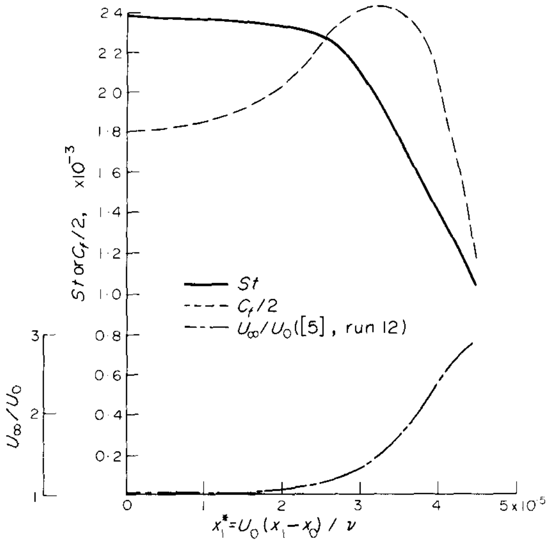

FIG. 1. Comparison of evolution of theoretical Stanton number and skin-friction coefficient in moderately short highly accelerated turbulent boundary layers.

RESULTS AND DISCUSSION

Equations (21), (22), (26) and (27) have been integrated numerically along streamlines to determine the evolution of several velocity and thermal boundary layers in severe pressure gradients. The numerical integrations were carried out by using an implicit method which was stable for all ratios of longitudinal to transverse increments. Because of the steep gradients close to the wall, more points were used in that region. The boundary conditions for equations (21) and (22) were $U_1 = 0$ and $T = T_w$ at the wall, $T = T_\infty$ at the edge of the thermal boundary layer, and $U_1 = U_\infty$ at the edge of the velocity boundary layer.

Initial values of Stanton number and the longitudinal pressure distributions for the severe favorable-pressure-gradient data of Moretti and Kays [5] were used, and the predicted results were compared with data from those experiments. Since initial velocity and temperature profiles and initial $\overline{u_1u_2}$ and $\overline{\tau u_2}$ distributions were not given in [5], they were calculated from the analysis of [11], together with initial Stanton and Reynolds numbers from [5] and skin friction coefficients from [12]. Although the analysis of [11] does not include wake regions for the profiles, reasonably good agreement with Klebanoff's flat-plate data is obtained by using a Karman constant of 0·36 in the logarithmic equation and applying that equation out to the edge of the boundary layer ([12], Fig. 1). It was shown in [1] that the wake region is quickly destroyed by strong favorable pressure gradients, such as the ones considered here. It might be emphasized that the analysis of [11] and [12] is not a part of the present theory, but is used only as a device for obtaining initial conditions.

Figure 1 shows theoretical Stanton-number and skin-friction-coefficient variations with dimensionless longitudinal distance for run 12 from [5]. The shear stress and heat transfer at the wall for the theoretical curves were obtained from the slopes of the velocity and temperature profiles at the wall by using points very close to the wall ($\sqrt{(\tau_w/\rho)}x_2/\nu \ll 1$). Also included in the plot are experimental values for U_∞/U_0. Initial conditions were taken at $x_0 = 4{\cdot}32$ ft. The difference between the Stanton number and skin-friction-coefficient variations is rather striking and indicates that Reynolds analogy does not apply in regions of severe pressure gradients. This difference is also indicated in the experimental results of [2] and [5].

The effect of favorable pressure gradients on velocity and temperature distributions is illustrated in Fig. 2, where theoretical values of T^+ and U_1^+ are plotted against x_2^+ for a low and a high value of pressure-gradient parameter $K = -(\nu/\rho U_\infty^3)\,\mathrm{d}P/\mathrm{d}x_1$. The results are again for run 12 from [5]. The effect of the pressure gradient on the T^+ profile tends to be opposite to that on the U_1^+ profile. Whereas the pressure gradient flattens the U_1^+ profile in the outer region of the boundary layer, it steepens the T^+ profile in that region. The same trends have been observed experimentally in [13] and [3]. The difference between the velocity and temperature results (or the skin friction and heat transfer results) is evidently due to the fact that the equation for the evolution of the mean velocity

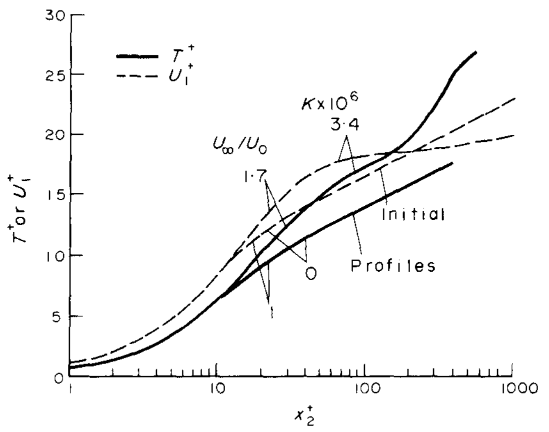

FIG. 2. Comparison of evolution of theoretical velocity and temperature distributions in moderately short highly accelerated turbulent boundary layers.

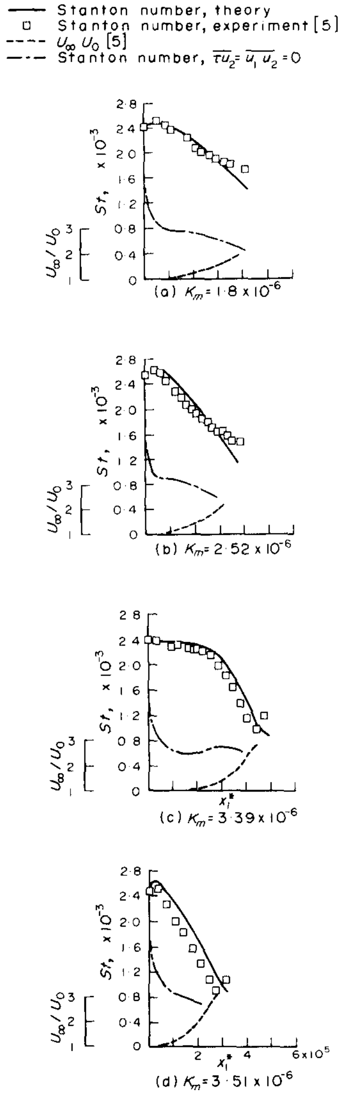

FIG. 3. Comparison of theory and experiment for evolution of Stanton number in moderately short highly accelerated turbulent boundary layers.

contains a pressure-gradient term [equation (9)] whereas that for the mean temperature does not [equation (3)]. Although the temperature equation does not contain a pressure gradient term, the pressure gradient can still affect the temperature through the mean velocity.

A comparison between theory and experiment for the evolution of Stanton number in severe favorable pressure gradients is presented in Fig. 3 for four values of maximum pressure–gradient parameter K_m. The values of x_0 for the initial conditions for the four runs (runs 41, F-3, 12, and 7) were, respectively, taken at $x_0 = 3{\cdot}64$, 3·64, 4·32 and 2·44 ft. For the smaller values of K_m good agreement between theory and experiment is indicated for values of U_∞/U_0 which are not too large. It appears that the range of values of U_∞/U_0 for which the theory applies increases as the pressure–gradient or acceleration parameter increases, as would be expected from equation (25). For values of x_1^* (or of U_∞/U_0) greater than those shown in Fig. 3, the theory appears to break down because the total normal strain $\ln U_\infty/U_0$ becomes too great and/or the pressure-gradient parameter K becomes too small.

Figure 3 also shows Stanton numbers calculated for turbulent initial velocity and temperature profiles, but for $\overline{\tau u_2} = \overline{u_1 u_2} = 0$. It is seen that the turbulent stresses and fluxes have a very large effect on the evolution of the Stanton number. The large effect of turbulence on Stanton number perhaps raises questions about calling regions such as those shown in Fig. 3 "relaminarization regions," although at the large values of K_m and x_1^* there is some tendency for the zero-turbulence curves to approach those for turbulence. The effects of turbulence on the velocity and temperature distributions are also considerable, although the general trends without turbulence are similar to those in Fig. 2.

It should, of course, be remembered that Stanton number, as well as C_f, T and U_1 at any longitudinal position depend on the whole distribution of pressure gradients up to that position. That is, Stanton number, for instance, is a functional of $\mathrm{d}P/\mathrm{d}x_1$, or

$$St = St\left[\frac{\mathrm{d}P}{\mathrm{d}x_1}(\xi)\right]$$

where $0 < \xi < x_1$.

CONCLUSIONS

The use of a Reynolds shear stress and turbulent heat transfer which remain frozen at their initial values as they are convected along streamlines gives results in agreement with experiment for severe favorable pressure gradients, when the total normal strain is not excessive. Both theory and experiment showed a flattening of the velocity profile in the outer region of the boundary layer, whereas they showed a steepening of the temperature profile in that region. Also, the evolution of the Stanton number differed considerably from that of the skin friction coefficient; Reynolds analogy between skin friction and heat transfer evidently does not apply in severe pressure gradients. Comparison of calculations with and without turbulent stresses and fluxes indicated that turbulence has a very large effect on the evolution of the Stanton number.

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EVOLUTION DU TRANSFERT THERMIQUE ET DE L'ECOULEMENT POUR DES COUCHES LIMITES TURBULENTES ASSEZ COURTES DANS DES GRADIENTS DE PRESSION SEVERES

Résumé—On analyse le développement initial et intermédiaire de couches limites dynamiques et thermiques fortement accélérées. Pour des accélérations (ou des gradients) suffisamment grandes et pour des contraintes normales totales pas trop fortes, les équations, pour les tensions de Reynolds et le transfert thermique turbulent, se simplifient et donnent des contraintes et des flux qui restent approximativement constants dans le sens des lignes de courant. Les résultats théoriques pour l'évolution du nombre de Stanton et la distribution de la vitesse moyenne et de la température en gradient de pression favorable s'accordent avec l'expérience.

ENTWICKLUNG DES WÄRMEÜBERGANGS UND DER STRÖMUNG IN KURZEN, TURBULENTEN GRENZSCHICHTEN BEI GROSSEN DRUCKGRADIENTEN

Zusammenfassung—Das Anlauf- und Zwischenstadium der Strömungs- und Temperaturgrenzschicht bei hohen Beschleunigungen wird untersucht. Für ausreichend große Beschleunigungen (oder Druckgradienten) und normale Gesamtbelastung vereinfachen sich die Gleichungen für Schubspannung und Wärmeübergang, so daß sich Spannungen und Wärmeströme ergeben, die längs Stromlinien näherungsweise konstant bleiben. Die theoretischen Ergebnisse stimmen in Stanton-Zahl und mittlerer Geschwindigkeits- und Temperaturverteilung bei günstigen Druckgradienten mit den Experimenten in den betrachteten Fällen überein.

РАЗВИТИЕ ТЕПЛООБМЕНА И ТЕЧЕНИЯ В ТУРБУЛЕНТНЫХ ПОГРАНИЧНЫХ СЛОЯХ СРЕДНЕЙ ПРОТЯЖЕННОСТИ ПРИ БОЛЬШИХ ГРАДИЕНТАХ ДАВЛЕНИЯ

Аннотация — Анализируется начальная и промежуточная стадии развития сильно ускоренного динамического и теплового пограничных слоев. В случае достаточно больших ускорений (или градиентов давления) и суммарных нормальных деформаций, не являющихся избыточными, для определения напряжений и потоков, которые приблизительно остаются постоянными вдоль линии тока, упрощаются уравнения напряжения Рейнольдса и турбулентного теплообмена. В рассматриваемых случаях экспериментальные данные согласуются с теоретическими результатами об изменении числа Стантона и распределении средних скоростей и температур при наличии отрицательных градиентов давления.